## transformations

- [SQA] 1. The diagram shows a sketch of the function y = f(x).
  - (*a*) Copy the diagram and on it sketch the graph of y = f(2x).
  - (*b*) On a separate diagram sketch the graph of y = 1 f(2x).



Part	Marks	Level	Calc.	Content	Answer	U1 OC2
<i>(a)</i>	2	В	CN	A3	sketch	2009 P1 Q23
<i>(b)</i>	3	В	CN	A3	sketch	
•1 •2 •3 •4 •5	ic: scal ic: ann ss: corr ic: star ic: con	ling para totate gr rect orde rt to ann nplete ar	allel to > aph er for ref otate fir nnotatic	k-axis fl(x) and trans nal sketch on	<ul> <li>•<sup>1</sup> sketch and one of (0, (-2,8))</li> <li>•<sup>2</sup> remaining points</li> <li>•<sup>3</sup> reflect in <i>x</i>-axis the translation</li> <li>•<sup>4</sup> sketch and one of (0,1 (-2,-7))</li> <li>•<sup>5</sup> remaining points</li> </ul>	,0), (1,8), n vertical 1), (1,−7),

2. Part of the graph of y = f(x) is shown in the diagram. On separate diagrams sketch the graphs of

$$(a) \quad y=f(x+1)$$

$$(b) \quad y = -2f(x)$$

Indicate on each graph the images of O, A, B, C and D.



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Part	Marks	Level	Calc.	Content		Answer	U1 OC2
<i>(a)</i>	2	С	NC	A3			1999 P1 Q10
(b)	1	С	NC	A3			
<i>(b)</i>	2	A/B	NC	A3			
• <sup>1</sup> tr • <sup>2</sup> p A	ranslation positions o A, B, C, D, C he sketch	of $\begin{pmatrix} -1\\ 0 \end{pmatrix}$ f images of clear from	of t		•3 •4 •5	reflect in x - axis double y - coordinates positions of images of A, B, C, D, O clear from the sketch	

- 3. The diagram shows a sketch of part of the graph [SQA] of  $y = \log_2(x)$ .
  - (*a*) State the values of *a* and *b*.
  - (*b*) Sketch the graph of  $y = \log_2(x+1) 3$ .



Part	Marks	Level	Calc.	Content	Answer U1 OC2
<i>(a)</i>	1	A/B	CN	A7	a = 1, b = 3 2001 P1 Q10
<i>(b)</i>	3	A/B	CN	A3	sketch
• <sup>1</sup> • <sup>2</sup> • <sup>3</sup> • <sup>4</sup>	pd: use evaluate ss: use ic: iden ic: iden	$e \log_p q$ $e \log_p p^k$ a transl ntify one ntify a se	$= 0 \Rightarrow$ ation e point econd p	q = 1 and oint	<ul> <li><sup>1</sup> a = 1 and b = 3</li> <li><sup>2</sup> a "log-shaped" graph of the same orientation</li> <li><sup>3</sup> sketch passes through (0, -3) (labelled)</li> <li><sup>4</sup> sketch passes through (7,0) (labelled)</li> </ul>

[SQA]

[SQA] 4. (a) Express  $f(x) = x^2 - 4x + 5$  in the form  $f(x) = (x - a)^2 + b$ .

- (*b*) On the same diagram sketch:
  - (i) the graph of y = f(x);
  - (ii) the graph of y = 10 f(x).
- (c) Find the range of values of x for which 10 f(x) is positive.

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
<i>(a)</i>	2	С	NC	A5	a = 2, b = 1	2002 P1 Q7
<i>(b)</i>	4	С	NC	A3	sketch	7
(C)	1	С	NC	A16, A6	-1 < x < 5	
•1 •2 •3 •4 •5 •6 •7	pd: pro square pd: pro square ic: inte ic: inte ss: refle ss: trar ic: inte	ocess, e ocess, e erpret m erpret y-: ect in x nslate pa erpret gr	.g. co .g. co inimum intercep axis rallel to aph	mpleting the mpleting the ot oy-axis	<ul> <li><sup>1</sup> a = 2</li> <li><sup>2</sup> b = 1</li> <li><sup>3</sup> any two from: parabola; min. t.p. (2, 1)</li> <li><sup>4</sup> the remaining one from</li> <li><sup>5</sup> reflecting in <i>x</i>-axis</li> <li><sup>6</sup> translating +10 units, <i>y</i>-axis</li> <li><sup>7</sup> (-1,5) i.e1 &lt; x &lt; 5</li> </ul>	; (0,5) above list parallel to

[SQA] 5. The sketch shows the graph of y = f(x) for  $-2 \le x \le 4$ . The function g(x) has the line x = 4 as an axis of symmetry and g(x) = f(x) for  $-2 \le x \le 4$ .

On separate sketches indicate

- (a) y = g(x) for  $-2 \le x \le 10$
- (b) y = -2g(x) for  $0 \le x \le 8$



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Part	Marks	Level	Calc.	Content	Answer	U1 OC2
<i>(a)</i>	2	С	CN	A3		1992 P1 Q10
(b)	2	С	CN	A3		]
• <sup>1</sup> fr • <sup>2</sup> fr - corr - zerc - min - ann	or any two or the other rect shape a os at 6 and 1 imum at (8 otation	from list two nd range 10 ,-3)	Ź		<ul> <li><sup>3</sup> for any two from list</li> <li><sup>4</sup> for the other two</li> <li>- correct shape and range</li> <li>- zeros at 2 and 6</li> <li>- extremes at (0,6), (8,6), (4,-6)</li> <li>- annotation</li> </ul>	

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[SQA] 6. The diagram shows the graph of y = f(x). Sketch the graph of y = 2 - f(x).



Part	Marks	Level	Calc.	Content	Answer	U1 OC2
	1	C	NC	A3		1993 P1 Q8
	2	A/B	NC	A3		
• <sup>1</sup> r	eflection is ranslation wo trans.	in Ox $ \begin{pmatrix} 0 \\ 2 \end{pmatrix} $ in correct	order, a	nnotate diagram	3	7

[SQA] 7. Part of the graph of y = f(x) is shown in the diagram. On separate diagrams sketch the graphs of

(i) 
$$y = f(x-1)$$

(ii) 
$$y = -f(x) - 2$$

indicating on each graph the images of A, B, C and D.



Part	Marks	Level	Calc.	Content	Answer	U1 OC2
<i>(i)</i>	2	С	CN	A3		1996 P1 Q8
(ii)	3	A/B	CN	A3		
•1	translatio	n of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$		• <sup>5</sup> $A(2,-2) B(0,$	-4) C(-1,-5) D(-3,-2)	ut a
•2	A(3,0) B	(1,2) C(0	,3) D(-2	2,0)		
•3	reflect in	x – axis				2
•4	translatio	n of $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	)			3 -3

[SQA] 8. The diagram shows a sketch of the parabola y = f(x).



(4)

(2)

(3)

- (a) Copy the sketch of y = f(x). On your diagram, draw the parabola with equation y = -f(x) + 3.
- (b) State the values of x for which  $3 f(x) \ge 0$ .
- (c) If g(x) = 3 f(x), express g(x) in terms of x.

Part	Marks	Level	Calc.	Content	Answer	U1 OC2		
( <i>a</i> )	4	A/B	NC	A3		1989 P2 Q4		
(b)	2	A/B	NC	A1				
(C)	3	A/B	NC	A7				
(a)	<ul> <li>(a) •<sup>1</sup> inverted shape</li> <li>•<sup>2</sup> passing through origin</li> <li>•<sup>3</sup> annotating (1,2)</li> <li>•<sup>4</sup> annotating (2,0)</li> </ul>							
(b)	b) $\bullet^5$ endpoints of $0 \le x \le 2$ $\bullet^6$ "less than signs" of $0 \le x \le 2$							
(c)	• <sup>7</sup> $g(x)$ • <sup>8</sup> (1,2) • <sup>9</sup> $g(x)$	$= ax(x - x)$ $\Rightarrow 2 = a(x - x)$ $= -2x(x - x)$	2) (1 – 2) – 2)					

[SQA] 9. Make a copy of this graph of  $y = \log_{10} x$ . On your copy, sketch the graph of  $y = \log_{10}(x-2)$ .





[SQA] 10. A sketch of the graph of y = f(x) where  $f(x) = x^3 - 6x^2 + 9x$  is shown below. The graph has a maximum at A and a minimum at B(3,0).



- (*a*) Find the coordinates of the turning point at A.
- (*b*) Hence sketch the graph of y = g(x) where g(x) = f(x+2) + 4.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes.

(c) Write down the range of values of k for which g(x) = k has 3 real roots.

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1	

Part	Marks	Level	Calc.	Content		Answer	U1 OC3
<i>(a)</i>	4	С	NC	C8		A(1,4)	2000 P1 Q2
<i>(b)</i>	2	С	NC	A3		sketch (translate 4 up, 2	
						left)	
(C)	1	A/B	NC	A2		4 < k < 8	
•1 •2 •3 •4 •5 •6 •7	ss: knc pd: diff ss: knc pd: pro ic: inte ic: inte	eventiate ow gradi ow gradi erpret tra erpret tra erpret sk	ferentia e correction $= 0$ ansform ansform	ite tly nation nation	tra	•1 $\frac{dy}{dx} = \dots$ •2 $\frac{dy}{dx} = 3x^2 - 12x + 9$ •3 $3x^2 - 12x + 9 = 0$ •4 $A = (1, 4)$ anslate $f(x)$ 4 units up, 2 units •5 sketch with coord. of A'(- •6 sketch with coord. of B'(1) •7 $4 < k < 8$ (accept $4 \le k \le 1$	s left -1,8) ,4) 8)

- The diagram shows a sketch of part of the graph of  $y = \log_5 x$ . 11. [SQA]
  - (a) Make a copy of the graph of  $y = \log_5 x$ . On your copy, sketch the graph of  $y = \log_5 x + 1$ . Find the coordinates of the point where it crosses the *x*-axis.





On your copy, sketch the graph of  $y = \log_5 \frac{1}{x}$ .

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
<i>(a)</i>	2	С	NC	A32		1994 P1 Q16
<i>(a)</i>	1	A/B	NC	A3		
<i>(b)</i>	2	A/B	NC	A28, A3		
•1	sketch of	f new fun	iction 1		•4 $\log_5 \frac{1}{x} = -\log_5 x$	1./
•2	$\log_5 x + \frac{1}{2}$	1 = 0	( <u>1</u> ,@	11	• <sup>5</sup> reflect in $x - axis$	0 1.00
•3	$\left(\frac{1}{5},0\right)$		0	(1, 0)		

12. The diagram shows a sketch of part of the graph of  $y = a^x$ , a > 1. [SQA] (a) If (1, t) and (u, 1) lie on this curve, write down the values of t and u.

- (b) Make a copy of this diagram and on it sketch the graph of  $y = a^{2x}$ .
- (c) Find the coordinates of the point of intersection of  $y = a^{2x}$  with the line x = 1.



Part	Marks	Level	Calc.	Content	Answer		U3 OC3
<i>(a)</i>	2	С	CN	A2			1997 P1 Q19
(b)	2	A/B	CN	A3			
(C)	1	С	CN	A6			
• <sup>1</sup> $t=a$ • <sup>2</sup> $u=0$ • <sup>3</sup> both passing thr' same point on y-axis • <sup>4</sup> $y=a^{2x}$ starting below $y=a^{x}$ and finishing above • <sup>5</sup> $(1,a^{2})$ For mark 3 For mark 4							